1 Introduction

My areas of research interests are probability, data science, functional analysis and infinite dimensional analysis. My main research has been on my dissertation: taking the integral with respect to a normalized surface area measure of slices of the large N sphere $S^{N-1}(\sqrt{N})$ with an affine plane and showing that the limit, as N goes to infinity, results in a Gaussian integral on the affine subspace in infinite dimensions.

Next I am focusing on the construction of a natural basis of $L^2(S^{N-1}, \sigma)$ that will lead to a basis for the infinite dimensional space $L^2(R^\infty, \mu)$ where μ is the infinite Gaussian measure. This result has already been proven by using a limit of the spherical Laplacian and considering spherical harmonics however we focus on a construction utilizing orthogonal polynomials formed from the spherical and Gaussian inner products.

I am also interested in research in data science. I have been working on a project in topological data analysis on large medical data sets and am also interested in work on image analysis. I would like to learn more and pursue research in areas of compressed sensing and image detection as well.

Further explanation of these problems and future work as well as other interests are detailed below.

2 The Gaussian limit for high dimensional spherical means.

The Radon transform of a function f on $\mathbb{R}^N$ associates to each affine space A in $\mathbb{R}^N$ the integral of $\int_A f d\lambda$ where λ is the Lebesgue measure in $\mathbb{R}^N$. In an infinite dimensional setting [3] constructs the Gaussian Radon transform of a function ϕ on a Banach space B which associates to each closed affine space A a Gaussian integral of ϕ over A. Utilizing the relationship between high dimensional spherical surface measures and Gaussian measures (see for instance [1]) and following the direct predecessor of our work [5], we can further associate to a function f defined on A the integral $\int_{S_{A_N}} f d\bar{\sigma}$ where S_{A_N} is intersection of the sphere $S^{N-1}\sqrt{N}$, the sphere in $\mathbb{R}^N$ of radius $\sqrt{N}$ centered at the origin, with the affine space A. Now we show that in the large N limit these spherical integrals $\int_{S_{A_N}} f d\bar{\sigma}$ is the Gaussian Radon transform.

More formally, let A denote an affine subspace in l^2 of finite codimension and let S_{A_N} be the 'circle' of intersection of $A_N = A \cap \mathbb{R}^N \times \{0\}$ with the sphere $S^{N-1}(\sqrt{N})$:

$$S_{A_N} = A_N \cap S^{N-1}(\sqrt{N}). \tag{2.1}$$

We have proven the following theorem which encapsulates the main idea of the paper. We follow this by proving a version for more general Hilbert and Banach spaces.

Theorem 2.1. *Let L be a finite-codimension affine subspace in l^2. Let k be a positive integer; suppose that the image of L under the coordinate projection $l^2 \to \mathbb{R}^k : z \mapsto z_{(k)} = (z_1, \ldots, z_k)$ is all of $\mathbb{R}^k$. Let ϕ be a bounded Borel function on $\mathbb{R}^k$. Then*

$$\lim_{N\to\infty} \int_{S_{L_N}} \phi(x_1, \ldots, x_k)\, d\overline{\sigma}(x_1, \ldots, x_N) = \int_{\mathbb{R}^\infty} \phi(z_{(k)})\, d\mu(z), \tag{2.2}$$

where $\overline{\sigma}$ is the standard surface area measure on S_{L_N} (defined in (2.1)) normalized to unit total mass, and μ is the probability measure on $\mathbb{R}^\infty$ specified by the characteristic function

$$\int_{\mathbb{R}^\infty} \exp\left(i\langle t, x\rangle\right)\, d\mu(x) = \exp\left(i\langle t, z^0\rangle - \frac{1}{2}\left\|P_0 t\right\|^2\right) \qquad \text{for all } t \in \mathbb{R}_0^\infty, \tag{2.3}$$

where z^0 is the point on L closest to the origin and P_0 is the orthogonal projection in l^2 onto the subspace $L - z^0$.

Future work. Currently we are working on a generalization of the main result (Theorem 2.1) for a function ϕ which is only integrable with respect to Gaussian measure on $\mathbb{R}^k$. The next consideration is whether a similar result would be possible for an affine space with infinite codimension. This result is under consideration as certain important lemmas require finite dimensionality as a key element. Lastly there were many interesting results concerning projections on dense nested sequences of subspaces in infinite dimensional space that are being considered for an expository paper.

3 Gaussian Orthogonal Polynomials.

Following from work done in [4], we see that the inner product $L^2(S^{N-1}(\sqrt{N}), \overline{\sigma})$ limits to an inner product on $L^2(\mathbb{R}^\infty, \mu)$:

Theorem 3.1. *Suppose p and q are polynomial functions on $\mathbb{R}^k$, viewed also as functions on $\mathbb{R}^N$ for $N > k$ in terms of the first k coordinates. Then*

$$\lim_{N\to\infty} \langle p, q\rangle_{L^2(S^{N-1}(\sqrt{N}), \overline{\sigma})} = \langle p, q\rangle_{L^2(\mathbb{R}^\infty, \mu)}, \tag{3.1}$$

where

$$\langle p, q\rangle_{L^2(S^{N-1}(\sqrt{N}),\overline{\sigma})} = \int_{S^{N-1}(\sqrt{N})} p(x)\overline{q(x)}d\overline{\sigma(x)}$$

and

$$\langle p, q\rangle_{L^2(\mathbb{R}^\infty,\mu)} = \int_{\mathbb{R}^\infty} p\overline{q}d\mu.$$

Using the inner product to orthogonalize monomials on the sphere leads to the Hermite polynomials as a basis for $L^2(\mathbb{R}^\infty, \mu)$. This result has been shown in [7], following the works of [2], through different methods using the limiting behavior of the spherical Laplacian. Our work however uses a more algebraic framework through polynomials and their restriction as functions to spheres of varying radius and dimension.

Future work. There is no immediate future work planned for this particular paper but I am interested in further study of spherical harmonics and perhaps a closer look at other function spaces.

3.1 Data Science Project and Other Research Interests.

Besides work with Gaussian and spherical surface measures I am interested in expanding my research to data science. In particular I am currently working on a project with the UCONN Center for Quantitative Medicine in the area of data analysis. More specifically we have begun looking at topological data analysis techniques, such as the Mapper algorithm [6], to analyze medical data utilizing R and Postgres (SQL) programming language. I have also attended a MSRI summer school on topics of compressed sensing and sparsity and would like to pursue future work in these areas of Data Science.

References

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